Teaching Reading in Mathematics
2nd Edition

A Supplement to
Teaching Reading in the Content Areas
Teacher’s Manual (2nd Edition)

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Teaching Reading in Mathematics, 2nd Edition
A Supplement to Teaching Reading in the Content Areas: If Not Me, Then Who?
2nd Edition
MARY LEE BARTON
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Rationale

As this cartoon suggests, part of the challenge of teaching reading in mathematics stems from confusion over what “reading mathematics” actually means. Is it being able to read expressions with mathematical symbols? Is it being able to comprehend printed numerical data? Or is it being able to comprehend worded passages in, say, a mathematics textbook?

In this supplement to the manual Teaching Reading in the Content Areas: If Not Me, Then Who? (TRCA Teacher’s Manual), “reading mathematics” means the ability to make sense of everything that is on a page — whether the page is a worksheet, a spreadsheet, an overhead transparency, a computer screen, or a page in a textbook or journal — in other words, any resource that students might use to learn and apply mathematics.

Why should students learn to read mathematics text? Martinez and Martinez (2001) in Reading and Writing to Learn Mathematics discuss what happens when children read and write mathematics:

For starters, their learning incorporates some key ideas in the National Council of Teachers of Mathematics new Principles and Standards for School Mathematics (NCTM, 2000). They learn to use language to focus on and work through problems, to communicate ideas coherently and clearly, to organize ideas and structure arguments, to extend their thinking and knowledge to encompass other perspectives and experiences, to understand their own problem-solving and thinking processes as
well as those of others, and to develop flexibility in representing and interpreting ideas. At the same time, they begin to see mathematics, not as an isolated school subject, but as a life subject — an integral part of the greater world, with connections to concepts and knowledge encountered across the curriculum (see the Process Standards Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). (p. 5)

A second reason students need to learn how to read mathematics is that reading mathematics requires unique knowledge and skills not taught in other content areas. For instance, mathematics students must be able to read not only from left to right, as they do in other subject areas, but also from right to left (as when reading an integer number line), from top to bottom or vice versa (as when reading from tables), and even diagonally (when reading some graphs).

Third, mathematics texts contain more concepts per word, per sentence, and per paragraph than any other kind of text (Brennan & Dunlap, 1985; Culyer, 1988; Thomas, 1988). In addition, these concepts are often abstract, so it is difficult for readers to visualize their meaning.

Fourth, authors of mathematics texts generally write in a very terse or compact style. Each sentence contains a lot of information, and there is little redundancy. Sentences and words often have precise meanings and connect logically to surrounding sentences. Students who want to read mathematics texts quickly — as they might a short story in their language arts class — may miss significant details, explanations, and the underlying logic.

Mathematics also requires students to be proficient at decoding not only words but also numeric and nonnumeric symbols. Consequently, the reader must shift from “sounding out” words such as plus or minus to instantly recognizing their symbolic counterparts, + and –.

Even the layout of a mathematics text can inhibit comprehension. Students often scan a page of text looking for examples, graphics, or problems to be solved, skipping worded passages filled with crucial information.

Further, many mathematics textbooks are written above the grade level for which they are intended. Therefore, the vocabulary and sentence structure in a mathematics textbook are often especially difficult for the students using these texts.
Finally, publishers of many mathematics textbooks are including longer passages of prose — verbal text — and students and teachers alike need to understand how to navigate these passages successfully. Exhibit 1 illustrates this apparent trend in mathematics texts. Taken from a recently published textbook, *Discovering Algebra: An Investigative Approach* (Murdock, Kamischke, & Kamischke, 2000), the text on this page (and throughout the book) is markedly different from other, more “traditional” mathematics text pages, which typically contain short verbal passages, a few examples, and a set of problems for students to solve.

Exhibit 1. Lesson on Fractals

**Lesson 0.1: The Same yet Smaller**

In this lesson you will review some concepts related to fractional parts. You'll learn to draw a fractal design in the investigation, and then you will review how to add, subtract, multiply, and divide fractions.

**Investigation 0.1.1: Sierpinski Triangle and Fractional Parts**

You can draw some interesting figures that have the characteristic that smaller parts added to the drawing look like the original. The procedures involved are very simple and, if you draw carefully you can create some beautiful designs. This investigation will introduce you to one such figure called the Sierpinski triangle.

You will need a pencil, a straightedge, and the isometric dot paper worksheet for this investigation.

**Part 1: Drawing the Sierpinski Triangle**

Look at the large triangle labeled Stage 0. In a fractal design, the Stage 0 figure is the starting shape. You'll make the same changes to this figure repeatedly to create the fractal design.

**Stage 1** Start with the triangle labeled Stage 1. To create the Stage 1 figure, draw line segments to connect the midpoints of each side of the triangle.

**Stage 2** Start with the triangle labeled Stage 2. Connect the midpoints to create the Stage 1 figure again. Next mark the midpoint of each side in each small triangle. Draw line segments to connect the midpoints in the upward-pointing triangles only.

**Stage 3** Start with the triangle labeled Stage 3. Re-create the Stage 2 figure in this triangle. Next mark the midpoint of each side in each of the smallest triangles. Draw line segments to connect the midpoints in the upward-pointing triangles only. Check to see that your Stage 3 figure looks like the one at right.

**Stage 4** If you want to create a Stage 4 figure, start with the large triangle in the lower half of the worksheet. First connect the midpoints of the large triangle, and continue connecting the midpoints of each smaller upward-pointing triangle at each new stage until you have 81 small upward-pointing triangles.

No wonder reading mathematics presents such unique challenges. As one experienced educator shares:

I have reached the conclusion that for my students to reach their potential as mathematicians, they must learn to comprehend mathematical texts, that is, texts constructed of numbers, abstract symbols, and — yes — words. It follows logically from this conclusion, then, that someone — mathematics teachers? — must teach them to do so. (Fuentes, 1998, p. 81)

Other mathematics educators agree. For example, Reehm and Long (1996) write:

Current recommendations for instruction in mathematics make the need for strategic reading of mathematics texts even more crucial than in the past. . . . There is a place and a need for skill development in reading for the purpose of understanding mathematics concepts. . . . The best place to teach the specific reading skills necessary for mathematics is in the mathematics classroom. . . . (pp. 35–36)

For these reasons — and for others that are explained in this supplement — we have written *Teaching Reading in Mathematics*. In this manual, we explore what literacy in mathematics involves. We also present suggestions and strategies teachers can share with their students to help them become more proficient in reading and communicating in mathematics.

Specifically, we

- examine what the research says about the role of the reader, the role of climate, and the role of text features in mathematics as well as their implications for instruction;

- present math-specific examples of the strategies included in the *TRCA Teacher’s Manual* so that mathematics teachers can see how to use and apply these strategies in their classes; and

- present additional strategies to help students become more proficient in reading mathematics.
Section 1
Three Interactive Elements of Reading

Introduction
As outlined in the second edition of the TRCA Teacher’s Manual, reading is a constructive process in which readers interact with text, using prior knowledge and experience to make connections, generate hypotheses, and make sense of what they read.

Too often, when students have trouble comprehending a textbook, teachers opt not to use the textbook and to teach mathematics concepts by lecturing about them. These teachers may think they are helping students by translating text material into verbal explanations, diagrams, and charts. However, “when we see these practices in mathematics classrooms, we see instructors in the process of constructing understandings for their students and then handing them over” (Fuentes, 1998, p. 82). In essence, students are deprived of the opportunity to make their own connections and to wrestle with ideas in order to make sense of them.

If students are to construct their own meaning from mathematics text, how can teachers guide and support this process?

Teaching students to comprehend mathematical text entails

- helping students assume their role as readers of mathematics,
- establishing a climate that is conducive to reading and learning mathematics,
• introducing students to the role that text features — vocabulary and text style — play in comprehension, and

• equipping students with strategies to learn new concepts and comprehend mathematics text content.
The Role of the Reader

Things to Think About

1. How do students’ experiences and prior knowledge of mathematics affect their learning?
2. How can teachers help students develop rich, organized knowledge structures and networks in mathematics?
3. How can teachers motivate students to learn and practice reading strategies?

“Don’t tell me why. Just tell me how,” the student urged, as the teacher tried to explain the reasoning behind how to solve a mathematics problem the student was working on in study hall. The student didn’t want to be bothered by what he considered “extra” input. He was eager to get the answer. Like many readers of mathematics text, he wasn’t eager to do the work of constructing meaning. He was more interested in finding a procedure he could use to get the correct answer than in learning how to explain a process or communicate discoveries.

Yet comprehending mathematics, like any other subject, is a constructive process. Research (e.g., Siegel & Borasia, 1992) has shown that “in order to acquire mathematical expertise in a durable and useful form, students need to construct mathematical knowledge and create their own meaning of the mathematics they encounter” (p. 19).

What roadblocks might prevent readers from constructing meaning when reading mathematics text? A number of potential pitfalls in readers’ prior knowledge and mental disposition can create difficulty when reading to learn mathematics.
Prior Knowledge

Two areas that can prevent learning from text are inadequate prior knowledge and prior knowledge that is not organized or accessible in long-term memory.

As discussed in the TRCA Teacher’s Manual, the extent of learners’ prior knowledge and experience has a direct effect on their acquiring new knowledge and skill. For example, the student who does not understand addition will be ill equipped to learn multiplication. Similarly, the student who never learns the “why” behind the “how” of solving certain mathematics problems will have a hard time applying skills used in one type of problem to other types.

This means that students who rely on algorithms alone to solve mathematics problems may find mathematics more difficult as they progress through their school years. Sometimes well-intentioned parents try to help children be more efficient by teaching them algorithms — tried-and-true ways to get answers using procedures like carrying and borrowing. These algorithms, devised as paper-and-pencil procedures, were designed to be quick and efficient. However, they often do not help people understand why they work. In fact, algorithms can make understanding more difficult (see a Mathematics for Parents Newsletter on Place Value (n.d.), Wisconsin Center for Education Research).

However, having a rich background in mathematics is not always a guarantee that a student will be able to solve more complex mathematics problems; the learner’s prior knowledge must also be organized and accessible in long-term memory. Alvarez and Risko (cited in Fuentes, 1998) assert that it is this organized body of knowledge that learners access while reading:

The richer and more organized a reader’s knowledge structures and networks are for a given topic, the better he or she will manipulate them in response to the text and the more likely that he or she will achieve appropriate understanding of the topic. (p. 82)
This means that in mathematics class, students must develop organized constructs that help them understand and explain how similar concepts and procedures are related to one another. If they do not, then they will have a difficult time recalling what they have learned and seeing how to apply procedures or concepts in a later chapter or a different context.

For example, one feature of many mathematics texts is that concepts are introduced but not discussed again for several chapters. As Smith and Kepner (1981) explain, mathematics concepts may be “developed in a spiral curriculum in which concepts, words, and symbols are developed and practiced, then followed by a period of disuse” (p. 10). When students merely memorize definitions for these concepts, words, and symbols and then the concepts appear again later in the text, teachers have to spend time re-teaching before beginning the lesson. However, if students develop a thorough and organized knowledge structure and network about this content, they will be able to recall and use that prior knowledge more quickly and effectively.

**Instructional Implications**

Teachers can use a number of strategies to help students acquire and access rich, well-organized knowledge structures and networks in mathematics.

First, students must recognize the importance of being able to activate and access their prior knowledge. As discussed in the *TRCA Teacher’s Manual*, teachers can show students how to activate their prior knowledge of a topic they will be studying by demonstrating basic pre-reading techniques such as

- brainstorming ideas that a topic brings to mind;
- previewing a passage, noting headings and bold print; and
- constructing a graphic organizer, web, or outline from passage headings for use in note taking.

Naturally, teachers also need to ensure that their students have a sufficient prior knowledge base before introducing new material.
Discovering what students already know about a topic helps teachers design instruction around the missing knowledge. A number of strategies can help teachers determine what students know before they begin studying a new topic:

- Semantic Mapping (p. 77)
- Word Sort (p. 86)
- Anticipation/Prediction Guide (p. 95)
- Knowledge Rating Chart (p. 108)
- K-W-L (p. 109)
- Problematic Situation (p. 116)
- Learning Log (p. 132)

Teachers also can help their students learn how to process, organize, and store new information in their long-term memory through the use of graphic organizers (see page 101). Graphic organizers are visual maps or representations that can describe how information in a chapter or a book is organized, or highlight the essential characteristics of a specific concept. Initially, teachers should model for students how to create and use a graphic organizer. For example, teachers may want to provide students with an advance organizer at the beginning of a new unit. Advance organizers include introductory outlines, maps, and webs that help students make connections between what they already know about a topic and what they will be learning. In addition, advance organizers provide students with a visual of the overall structure of key concepts and procedures that will be covered and how these fit together.

The first few advance organizers that a teacher shares with students should be teacher constructed. Eventually, the teacher can leave some spaces in the organizer for students to fill in as the class progresses through the unit. Exhibit 2 is an example of a partially completed advance organizer for a chapter on probability.
Exhibit 2. Advance Organizer for a Text Chapter on Probability

Comprehension Guide for Probability

Sample Space
(all possible Outcomes)

Counting Principles

Impossible

Fundamental Counting Principle

Combinations (unordered)

Permutations

e.g.,

e.g.,

Events

Probability

P (Event) =

Independent

Dependent

(likelihood)

Sample Space
(all possible Outcomes)

Counting Principles

Impossible

Fundamental Counting Principle

Combinations (unordered)

Permutations

e.g.,

e.g.,

Directions: Save this sheet. Fill in the definitions, diagrams, and examples as we work through Chapter 11. This will be a useful study guide.

Additional types of graphic organizers include semantic and concept maps (see pages 101–105 and 121–122 in this supplement). Again, teachers should model for students how to represent the essential elements of a concept and explain how these are related. Ultimately, students should practice constructing their own meaning of the text content or of a concept by creating their own ways of visually representing the information. And, by sharing a variety of graphic organizers on the same content, students can deepen their understanding of concepts.

In addition to helping students learn how to process, organize, and store new information in their long-term memory, regular use of these kinds of graphic organizers in the classroom can increase comprehension, retention, and recall of information (Jones, Palincsar, Ogle, & Carr, 1987). Moreover, constructing meaning through a visual organizer can challenge students to restructure misconceptions in their existing schema rather than distort new information to fit their beliefs (Fuentes, 1998).

**Mental Disposition**

Exemplary mathematics educators know that students’ attitudes about reading and learning mathematics affect their achievement. Of particular concern, then, are reports that students’ motivation to learn wanes over time. For example, Holloway (1999) notes that “intrinsic motivation for literacy and other academic subjects declines in middle school” (p. 80). What can teachers do to increase students’ motivation to learn from reading mathematics text?
In addition to connecting reading assignments to students’ real-world experiences, teachers need to show students that becoming effective consumers of mathematics text has value. Students need to see firsthand that practicing the right reading strategies will improve their achievement.

This is especially true of struggling readers. Some readers who struggle also have a poor attitude toward reading and often don’t see the connection between the effort they put forth to read and complete their assignments and the grades they earn in class. Marzano, Pickering, and Pollock (2001) cite a set of studies demonstrating that simply showing students that added effort improves their achievement actually increases students’ achievement. The authors note that since “students might not be aware of the importance of believing in effort,” teachers should “explicitly teach and exemplify the connection between effort and achievement” (p. 51).

**Instructional Implications**

To demonstrate to students how their effort affects their achievement, Marzano et al. (2001) suggest that students periodically assess their level of effort on assignments and track the impact of their effort on the grades they earn. Teachers can give students a set of effort and achievement rubrics (see Exhibit 3 on page 9), which students can use to assess and track their effort and achievement on a chart (see Exhibit 4 on page 10).

When students observe the impact that their effort and attitude have on their progress, they begin to see the value of applying reading strategies to improve their comprehension and learning. They also gain a sense of control over their learning — a crucial step in assuming more responsibility for their own learning.
### Effort and Achievement Rubrics for Mathematics

Scale: 4 = excellent; 3 = good; 2 = needs improvement; 1 = unacceptable

#### Effort Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>I worked on my mathematics assignment until it was completed. I pushed myself to continue working on the task even when difficulties arose, when a solution was not immediately evident, or when I had trouble understanding what an author was saying. I used obstacles that arose as opportunities to strengthen my understanding and skills beyond the minimum required to complete the assignment.</td>
</tr>
<tr>
<td>3</td>
<td>I worked on my mathematics assignment until it was completed. I pushed myself to continue working on the task even when difficulties arose, when a solution was not immediately apparent, or when I had trouble understanding what an author was saying.</td>
</tr>
<tr>
<td>2</td>
<td>I put some effort into my mathematics assignment, but I stopped working when difficulties arose, when a solution was not immediately evident, or when I had trouble understanding what an author was saying.</td>
</tr>
<tr>
<td>1</td>
<td>I put very little effort into my mathematics assignment.</td>
</tr>
</tbody>
</table>

#### Achievement Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>I exceeded the objectives of the assignment.</td>
</tr>
<tr>
<td>3</td>
<td>I met the objectives of the assignment.</td>
</tr>
<tr>
<td>2</td>
<td>I met a few of the objectives of the assignment, but didn’t meet others.</td>
</tr>
<tr>
<td>1</td>
<td>I did not meet the objectives of the assignment.</td>
</tr>
</tbody>
</table>
Exhibit 4. Effort and Achievement Chart

<table>
<thead>
<tr>
<th>Student</th>
<th>Assignment</th>
<th>Effort Rubric</th>
<th>Achievement Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greg Starek</td>
<td>Assignments are described in the text. Effort and Achievement Rubrics are not</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Assignment</th>
<th>Effort Rubric</th>
<th>Achievement Rubric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday, Sept. 21</td>
<td>Read the introduction to proportional reasoning. Conduct an investigation on body measurements: Record measurements in a table that includes ratios of certain measurements, make box plots of data from females and males in the class, and write a paragraph that compares the graphs.</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Wed., Sept. 23</td>
<td>Homework: Complete a problem set involving various ratios in several given data sets.</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Thurs., Sept. 24</td>
<td>Read about proportions, and conduct an investigation solving for an unknown in a proportion. Write an explanation of the process used in the investigation.</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Friday, Sept. 25</td>
<td>Homework: Complete a problem set on writing proportions and solving for an unknown quantity. Read about percents, and conduct an investigation on visualizing and computing percents.</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
The Role of Climate

The National Council of Teachers of Mathematics (2000) has articulated the important role that classroom climate plays in learning:

More than just a physical setting with desks, bulletin boards, and posters, the classroom environment communicates subtle messages about what is valued in learning and doing mathematics. Are students’ discussion and collaboration encouraged? Are students expected to justify their thinking? If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others’ arguments, then creating an environment that fosters these kinds of activities is essential. (p. 18)

Learning is most likely to occur when students see value in what they are doing, when they believe they can be successful, and when they feel safe. Unfortunately, some students do not see any value in learning mathematics. This is especially true for students who have a history of failure and experiences in which they were criticized or humiliated for taking risks. Perhaps this is a consequence of past mathematics instruction that focused mainly on product rather than on the process of doing mathematics.

Three Interactive Elements of Reading

Reader  Climate

Text  Features

Things to Think About

1. What does “climate” in the mathematics classroom include?
2. What can mathematics teachers do to create a classroom climate that supports learning?
Notes

We now know that mathematics education should emphasize active, flexible, and resourceful problem solving and should place greater emphasis on the affective dimensions of learning mathematics (National Research Council, 1989, 1990; NCTM, 1989, 1991, 2000). No longer should instruction focus on imitating and memorizing what is presented by the teacher, but rather on “students’ problem-solving strategies, including their ability to generate and define problems, as well as their mathematical reasoning and communication” (Siegel & Borasia, 1992, p. 19).

Instructional Implications

In addition to those suggestions given in the TRCA Teacher’s Manual, there are a number of things mathematics teachers can do to establish a classroom climate that supports learning. First, teachers should shift instruction to emphasize process. Point out to students that there may be more than one way to solve a problem or that, in some situations, there may be multiple solutions. Model for students how you reach a solution, but also let students discuss in groups the steps that they went through to reach their solutions. Another technique is to ask questions that allow for more than one response. For example, you might set up a word problem in which students are told that they have a certain amount of money to spend for camping supplies. Provide a list of items with prices marked, and allow students to “shop” so that they purchase what they think they will need for this scenario, spending an amount that uses their funds yet is within their budget. You might then ask students to write an explanation of their purchases and their calculations.

Another method of easing learners’ fears of making mistakes is to give students credit for the effort they put into solving a problem. Certainly, students are being asked to “show” or “explain” their work in class assignments and on assessments. Offer students credit for writing out their thought processes, so that they begin to value process too.
# Section 5
## Reading Strategies

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**Reflection Strategies** *(Questioning; Writing; Discussing)*

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M-4. List-Group-Label

What is it?
Similar to semantic mapping and word sort (pages 77 and 86 of this supplement), list-group-label helps students examine the relationships among subject-matter concepts. Taba, Durkin, Fraenkel, and McNaughton (1971) and Fraenkel (1973) note that this involves students in their own learning because they are responsible for contributing the vocabulary they associate with a particular concept rather than manipulating vocabulary provided by the teacher. As such, this strategy can activate prior knowledge and help learners make essential connections between their experience base and new understandings.

How could it be used in mathematics instruction?
This strategy is very effective in assessing students’ prior knowledge and classification skills. It may be used to involve students in reasoning about classifications and in making connections (relationships) among terms in selected categories.

How to use it:
1. Write a content-area term on the board or on an overhead transparency. Explain to students that this term has something to do with the next unit (or chapter).
2. Ask students to generate words and phrases that they associate with this term. As students volunteer responses, they will stimulate others in the class to contribute their ideas.
3. After you have developed a list of 15–30 words or phrases, ask students to consider what the words have in common and to organize them into categories. Remind them that these categories should identify significant relationships among the terms, and that the relationships should extend their learning. Grouping words by their initial letter, for example, is not an activity that identifies a mathematically significant relationship.
4. Once students have completed classifying these terms, ask them to explain the rationale behind their groupings. Small group work is likely to generate different categories, and class discussion of the differences can deepen understanding of the concepts.

5. Use this discussion as an opportunity to broaden students’ understanding of these concepts and how to apply this understanding when solving problems.

**Term: Measurement**

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M-22. Problematic Situation

What is it?
Problematic situation is a strategy that activates what students already know about a topic, motivates students to want to read the text, and helps them to focus on the main ideas presented in the text as they read. Developed by Vacca and Vacca (1993), it can be used with any text material dealing with a problem/solution relationship.

How could it be used in mathematics instruction?
This strategy can be used to engage students in mathematics problems that lead them to want to explore ideas through reading. It can also be used to evaluate students’ understanding of the mathematics content in a problem situation. This strategy is an effective way to bring real problem-solving contexts into mathematics.

How to use it:
1. Design a problematic situation similar to one presented in a selected text passage. Provide enough relevant information about the situation so students will be able to focus their attention on key ideas in the passage. Be sure to define clearly the context of the problem.

2. Pose the problem to students. Let cooperative groups generate and record possible results or solutions. When they have listed their solutions, let them discuss why each one is a good solution or why it would succeed.

3. Ask students to “test” their solutions when they read the assigned text material. Each group should refine or modify their solutions as they gain new information from their reading.

4. As a final activity, discuss with the class whether some of the students’ solutions might be better than those presented by the author.

For further discussion of this strategy, see the TRCA Teacher’s Manual, pp. 122–123.
Informational Text

Examples of Problematic Situations Involving Measurements and Percents

Copy Machine

The copy machine in Ms. Graber’s office has a zoom that allows the user to set the machine so that copies will be enlarged or reduced from the original image. The settings for this zoom feature are in percents. The office staff are in the habit of using the zoom feature simply by trying a number of settings until they find the one that fits their needs. But this results in a lot of wasted copies and discarded paper. Ms. Graber would like to give explicit directions to the staff so that they will learn how to set the zoom feature accurately the first time. How would you advise Ms. Graber to write these directions?

Package Design

Your firm has been selected to design a new package for the STARBAR Candy Company. The company is planning to reduce by 10% the size of the current candy bar it sells. The dimensions of the current unwrapped candy bar are 6 inches by 2 inches by 1 inch. The company must reduce the size to cut costs, but management recognizes that reducing the size may affect sales. Your task is to design the packaging so that it will minimize the appearance of the reduction in size. Write your response in the form of a proposal to the chairman of the board of STARBAR Candy Company.
M-31. Learning Log

What is it?
An effective means of writing-to-learn is keeping a learning log. Learning logs can foster reflection on reading processes and hands-on activities to increase students’ understanding. Learning logs differ from journals in that they focus on content covered in class, rather than on personal or private feelings. Students may reflect on how they feel, but it is always in relation to what is being studied in class. Santa and Havens (1991) suggest that teachers introduce learning logs to students as a way of writing down their thinking.

How could it be used in mathematics instruction?
Learning log entries may be incorporated across mathematics lessons. Writing activities can engage students in thinking about a concept and can help them examine more deeply the concept as they collect data or work with examples. Formulating explanations through writing helps students know if they really understand a concept. Writing can be used as a way to self-evaluate as students reflect on what they have learned.

How to use it:
1. Assign the topic. A learning log entry can be assigned at any time during class, depending upon the topic and your purpose.
2. Allow students “think time” to consider their response.
3. Give students time to write about the topic.
4. Encourage students to reread their learning log entries at a later date and reflect on how their ideas have changed.

For further discussion of this strategy, see the TRCA Teacher’s Manual, pp. 148–150.
Reflection Strategies

Learning Log Assignment Example

The following are possible learning log topics, adapted from Brudnak (1998).

Before learning — to activate and assess prior knowledge

- Why do we use rulers (or scales or other measuring devices)?
- What do these symbols mean?
- Describe instances when you use addition at home.
- How is multiplication similar to addition?
- Make a web to describe some uses of fractions.

During learning — to help students identify how well they understand what is being covered in class

- Explain how you know that $7 + 3 = 11 - 1$.
- How do you know what a story problem is asking you to do?
- Write a story problem in which you need to calculate $5 \times 7$.
- Find examples in our classroom of the geometric shapes we are studying.
- Draw three pictures that demonstrate the concept of multiplication.

After the lesson — to help students reflect on their learning

- I have trouble understanding. . . .
- Write a note to a student who was absent from class and explain what was learned in class today about right triangles.
- Write a note to your parents explaining how you know when a shape has a line of symmetry.
- My favorite kind of story problem is. . . .
- Explain how you could do the calculation $65 - 19$ in your head.
About the Authors

**Mary Lee Barton**, M.S. Ed., has worked in the areas of literacy, learning, and professional development for more than 25 years. She brings a wealth of practical classroom experience to her writing and professional development workshops. As a consultant for McREL, Barton coauthored *Teaching Reading in the Content Areas: If Not Me, Then Who?* and its supplements, *Teaching Reading in Mathematics, Teaching Reading in Science, and Teaching Reading in Social Studies*. Her articles “Addressing the Literacy Crisis: Teaching Reading in the Content Areas” and “Motivating Students to Read Their Textbooks” have appeared in the NASSP Bulletin. She has trained thousands of teachers and administrators across the country in content-area reading and writing instruction. Currently, Barton is a writer and a business and education consultant in private practice. She trains and provides technical assistance nationally to educators and business clients on literacy issues in education and in the workplace.

**Clare Heidema** worked for many years with the educational laboratories CEMREL and McREL, focusing on curriculum development, professional development, dissemination, and product development in mathematics education. She was principal author of the *Comprehensive School Mathematics Program* (CSMP) and served as director of the CSMP Developer/Demonstrator project for the National Diffusion Network from 1983 – 1996. She also served as a mathematics consultant for the Eisenhower Regional Consortium at McREL. Heidema is a former middle and high school mathematics teacher and has had teaching experience at all levels from elementary school to graduate school. She holds a BA/MA in mathematics, teaching certification from the University of Michigan, and has completed MS and doctoral requirements (ABD) in mathematics from Syracuse University. She has presented at numerous state, regional, and national conferences. Heidema currently works as an educational consultant.
Teaching Reading in Mathematics

TRAINING WORKSHOPS AVAILABLE

McREL delivers training and consultation on its series, *Teaching Reading in the Content Areas: If Not Me, Then Who?*, *Teaching Reading in Mathematics*, *Teaching Reading in Science*, and *Teaching Reading in Social Studies* to teachers, reading specialists, staff developers, and administrators.

These workshops focus on a framework for teaching reading in the content areas, and provide numerous strategies to help students better comprehend content-area reading material. Strategies emphasize students monitoring their own thinking, selecting appropriate strategies, and applying these strategies to increase their awareness and understanding of text.

The **Teacher Workshop** (designed for 4–12 educators) provides an overview of content-area reading instruction; engages participants in applying vocabulary, reading, and reflection strategies to specific content covered in their classrooms; and offers practical suggestions on integrating these strategies into existing curricula.

For more information about scheduling workshops and consulting services, contact McREL at 303-337-0990, or visit our website at [www.mcrel.org](http://www.mcrel.org).